Various measures of central Tendency

I Mathematical Averages :

(i) Airthmatic mean
(ii) Geometric Mean
(iii) Harmonic Mean
(iv) Quardratic Mean.

II Positional Averages :

(i) Median (M) (ii) Mode (Z)

III Commercial Averages :

(i) Moving Average
(ii) Progressive Average
(iii) Composite Average

Airthmatic Mean : The mean is the value obtained by adding the values of a set of observations divided by their number. Each dependent value plays an equal part in the determination of the mean.

Individual series :-

$$(\text{Mean})\ \overline{X} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$\boxed{\overline{X} = \frac{\Sigma x}{n}}$$

Discrete Series :-

$$\overline{X} = \frac{\Sigma fx}{N}$$

Continuous Series :-

$$\overline{X} = \frac{\Sigma fm}{N}$$

→ Methods of Calculation of Mean.

(a) Direct Method

$$\overline{X} = \frac{\Sigma X}{n}$$

(b) Short Cut Method

$$\overline{X} = A + \frac{\Sigma dx}{N}$$

(c). Step-deviation method

$$\overline{X} = A + \frac{\Sigma ds}{N} \times i$$

(d). Combined Mean

$$\overline{X} = \frac{N_1\overline{X}_1 + N_2\overline{X}_2 + N_3\overline{X}_3 + \ldots N_4\overline{X}_4}{N_1 + N_2 + N_3 + \ldots + N_n}$$

Median : Middle most central value of the series which distribute the series into two parts known as Median.

Properties of Median :

1. It is an average of position.
2. The sum of the deviations about the median, sign ignored, will be less than the sum from any other point.

→ Merits :

(1) The median is a very easy to calculate and is readily understood, specially in a series of individual observations and discrete series. In a series of individual observations, after arraying the values in a ascending or descending order and a in a discrete series, after making the frequencies cumulative median can be located just by observation.

(2) Unlike arithmetic mean the median can be determined where the data are incomplete, eg; irregular class-interval

or open-end class-interval series.

(3). It is not affected by the items on the extreme. It is independent of the range of the series of the spread of values above or below it.

(4). Median is an appropriate measure of central tendency in some qualitative characteristics which can be ranked such as intelligence, beauty etc

(5). Median can be located graphically also, while mean can not be

→ Demerits:

(1) The computation of median requires in certain cases arraying of the items. It is often a cumbersome job.

(2). The median and no. of a distribution are given then total value can not be known. Like arithmetic mean where $(\bar{X} \times N = \Sigma X)$, $M \times N = \Sigma X$ is not there.

(3). Being an average of position, median is not a mathematical concept suitable for further algebraic treatment. For ex: if one knows the medians of two distribution and the no. of items in each there is no algebraic way of combining these two figures to obtain the median of the combined distribution.

(4). It is not possible to obtain the actual median in the case of a group having an even no. of observations and thus in such a case it is a makeshift, an average of the two items in the middle is taken.

(5). A very little importance is attached to the items on the extremes and as such the median fails to register changes due to the changes in the values of the items on the extremes.

→ In Individual and discrete Series:-

$$M = \text{The size of } \left(\frac{N+1}{2}\right) \text{th term.}$$

→ In Continuous Series :-

$$M = l_1 + \frac{N/2 - C}{f} \times i \quad \Big| \quad M = l_1 + \frac{i}{f}(m - C)$$

Where, M = Median

l_1 = Lower limit of median class

i = class-interval of the median class

f = frequency of median class

c = Cumulative frequency of the class preceeding the median class.

m = $N/2$ th term.

Mode :→ The value that occurs most frequently in a statistical distribution.

OR, Mode is a position of greatest density or a point of highest concentration of value.

In Continuous Series :→

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

OR, $$Z = l_1 + \frac{f_1 - f_0}{(f_1 - f_0) + (f_1 - f_2)} \times i$$

→ **Merits** :-

(i). The mode can be found out for those quantitative data in which categorization or ranking is possible.

(ii). The mode has the great advantage because it is usually an actual value of an important part of the series, but not necessarily the major part.

(iii). It is not affected by extreme items. It can be calculated even if extremes are not known.

(iv). It is simple and precise.

(v). Mode is that point where there is more concentration of frequencies. Hence, it is best representation of data.

→ **Demerits** :-

(i) It is unsuitable in cases where relative importance of items have to be considered.

(ii). It does not lend itself to further mathematical treatment.

(iii). Choice of grouping has considerable influence on the value of the mode.

Relation Between Mean, Median and Mode

1. For Symmetric Distribution

$$\text{Mean} = \text{Median} = \text{Mode}$$

2. For +ve skewed frequency distribution

$$\text{Mean} > \text{Median} > \text{Mode}$$

3. For −ve skewed frequency distribution

$$\text{Mean} < \text{Median} < \text{Mode}$$

4. In Moderately Asymmetrical (skewed) frequency distribution relationship among mean, median and mode can be expressed as; $\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$

(i). $\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$

(ii) $\text{Median} = \frac{1}{3}(2\,\text{Mean} + \text{Mode})$

(iii). $\text{Mean} = \frac{1}{2}(3\,\text{Median} - \text{Mode})$.